Proposed quoting convention for HFF Icelandic Housing Bonds

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1 Real Cashflows

The Housing Bonds are index-linked amortising bonds, which pay a fixed real annuity that is adjusted for the CPI level on each payment date.

The annuity payment P to amortise a notional of N_0 with a single-period interest rate r is given by the standard mortgage formula

$$P = N_0 \frac{r}{1 - \left(\frac{1}{1+r}\right)^n} \tag{1}$$

for n amortisation payments. The interest rate r is given by the annual interest rate c of the bond (equivalent to the coupon of a normal bullet bond) through the formula

$$r = \frac{c}{f} \tag{2}$$

for a payment frequency of f (note that this is more a definition of c than a definition of r). The new bonds pay semiannual coupons, so $f = 2$.

Given the amortisation payments P and the interest rate r, it is possible to work out the remaining notional N_k at any one point in time. We find

$$N_k = N_0 \left[(1+r)^k - \frac{(1+r)^k - 1}{1 - \left(\frac{1}{1+r}\right)^n} \right] \tag{3}$$

for the remaining principal after k payments. The intuitive interpretation of this equation is that the first term on the right-hand side is the forward value of the original debt, while the second term is the forward value of the annuity payments already made. Naturally, this equation yields $N_k = N_0$ for $k = 0$ and $N_n = 0$.

The interest component paid on payment k is given by the product of the remaining principal after payment $k - 1$ and the interest rate r, i.e.,

$$I_k = N_0 \frac{r\left[(1+r)^n - (1+r)^{k-1}\right]}{(1+r)^n - 1} \tag{4}$$

while the amortisation component is simply the difference between the fixed payment P and the interest component I_k, i.e.,

$$A_k = N_0 \frac{r(1+r)^{k-1}}{(1+r)^n - 1} \tag{5}$$

Naturally, $P = I_k + A_k$ for all k.

2 Indexation

The new bonds are indexed to the Icelandic CPI. While Icelandic law describes the indexation in terms of adjusting the principal N_k for inflation, it may be easier to use the equations above and simply think of the cashflows P being adjusted. In this way, the amortisation schedule can be thought of as fixed when calculating real yields.

The adjustment is done by multiplying P with the index factor I_t

$$I_t = \frac{CPI_t}{CPI_0} \tag{6}$$

where CPI_0 is the base CPI for the bond and CPI_t is the index level applicable at the time of payment t. Usually, CPI_0 is chose so that the index ratio I is 1 on the first settlement day, but that is purely a matter of convenience.

Because CPI numbers are only published monthly, CPI_t itself needs to be calculated and in Iceland this is done in two different ways based on the CPI level assumed to have prevailed at the first day of the relevant month, CPI_m. If the next month's CPI level CPI_{m+1}, is already known (from around the 12th of the month), CPI_t is given by

$$CPI_t = CPI_m \left(\frac{CPI_{m+1}}{CPI_m}\right)^{\frac{d}{30}}, \tag{7}$$

otherwise the Central Bank of Iceland's inflation forecast for the annualised inflation rate ν is used and the calculation is:

$$CPI_t = CPI_m(1+\nu)^{\frac{d}{360}}. \tag{8}$$

In both cases, d is the number of days since the first day of the month, i.e., $d = 0$ on the first day, $d = 1$ on the second, etc.

3 Existing price quotations

Currently, IBH and IBN issues are quoted in two different conventions, but both are based on an inflation-adjusted principal amount of debt. This quoting convention means that the impact of inflation on prices is intermingled with that of interest accrual and yield changes, which is somewhat unsatisfactory. Ideally, the quoted bond price would move only as a result of yield changes. Further, the price should be in a clear relationship to the expected cashflows.

For the simpler of the two bond classes, the IBN issues, the market price M for real yield y is given by

$$M^{\mathrm{IBN}} = 100 * \frac{CPI_t}{CPI_0}\frac{P}{N_0}\sum_i \frac{1}{(1+y)^{t_i}} \tag{9}$$

where P is calculated according to equation (1), y is the market yield, and t_i is the year fraction from settlement to the cashflow date i, measured in a 30/360 daycount convention. For the first settlement day of the bond, this formula will yield a cash price of 100 if $CPI_t = CPI_0$ on that day and the yield is equal to the coupon. The price can therefore be interpreted as a percentage of the original debt amount. On the last coupon date,

$$M^{\mathrm{IBN}} = 100 * \frac{CPI_t}{CPI_0}\frac{P}{N_0} \tag{10}$$

so if inflation is disregarded, the price will gradually drop to the annuity payment.

The IBH issues are priced differently. Their market price is given by

$$M^{\mathrm{IBH}} = 100 * \frac{CPI_t}{CPI_0} \frac{r(1+r)^{k-1}}{1+\left(\frac{1}{1+r}\right)^n} \sum_i \frac{1}{(1+y_c)^{t_i}} \tag{11}$$

where k is the number of the next payment, i.e., $k = 1$ in the first interest period.

The yield y_c is the compounded yield $y_c = (1 + y/f)^f - 1$ with the coupon frequency $f = 4$. In effect, the price is quoted relative to the forward value of the original debt, i.e., the first term on the right hand side of equation (3).

4 Proposed price quotation

We propose that the market price be calculated in a way that keeps it close to par as long as yields stay at par. The natural way to achieve this is to quote the price of the bond as a percentage of the price the bond would have if it was trading at its coupon rate. Naturally, that would be a rather complicated formula. Instead, we propose to quote the bonds as percentage of their remaining outstanding principal, minus accrued interest. We also propose not to include the inflation uplift in the price.

We start from the general statement that the dirty price before adjustment for inflation is equal to the discounted value of the remaining cashflows, so

$$PV = P \sum_i \frac{1}{(1+y_c)^{t_i}} \tag{12}$$

The proposed pricing convention is to quote the present value minus the accrued interest as a fraction of remaining notional:

$$M^{\mathrm{HFF}} = \frac{PV - I_k \frac{df}{360}}{N_{k-1}} \tag{13}$$

where k is the number of the next coupon payment, i.e., $k = 1$ during the first interest period.

This equation can be simplified because I_k is rN_{k-1}. Also writing P and N_k as a function of N_0 using equations (1) and (3), we arrive at

$$M^{\mathrm{HFF}} = 100\% * \left[\frac{r(1+r)^n}{(1+r)^n - (1+r)^{k-1}} \sum_i \frac{1}{(1+y_c)^{t_i}} - r\frac{df}{360} \right] \tag{14}$$

where d is the number of days since the last coupon assuming 30 day months and $y_c = (1 + y/f)^f - 1$ as above.

If a trade is struck at the price M^{HFF} as described above, the invoice price will not simply be the product of price and notional amount as is currently the case (but note that the meaning of 'nominal amount' is different between IBH and IBN issues). Instead, the market price will have to be adjusted for accrued interest, remaining notional, and inflation. Below, we assume that the market convention will be to agree trades on the basis of a specified amount of original notional amounts because this will be the basis for the definitive notes should they ever have to be issued. In total, the invoice amount B for a notional amount N_0 will be

$$B = \frac{CPI_t}{CPI_0} N_k \left(M + r\frac{df}{360} \right) \tag{15}$$